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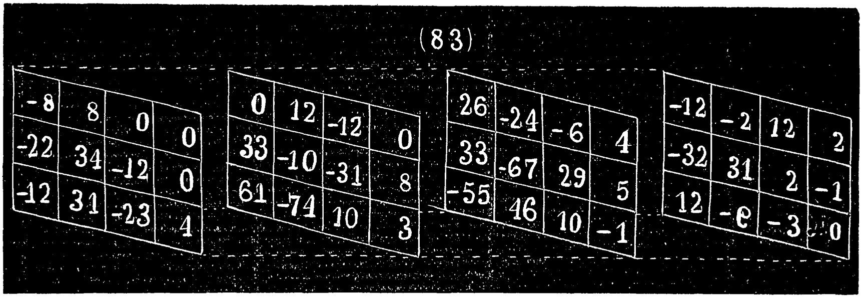


(83)

-8	8	0	0
-22	34	-12	0
-12	31	-23	4

0	12	-12	0
33	-10	-31	8
61	-74	10	3

26	-24	-6	4
33	-67	29	5
-55	46	10	-1

-12	-2	12	2
-32	31	2	-1
12	-e	-3	0

those in the two factors, so that the centre of forces here coincides with the —67. Estimating $b_{m,n,p}$ from this centre as an origin, we shall find that it reduces to zero for the same sets of values of m, n, p as it did in the cases of the two factors.

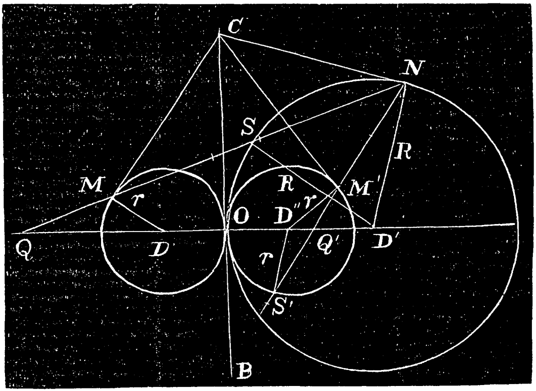
SOLUTION OF A SPECIAL CASE OF PROBLEM 288, *WHERE THE CIRCLES ARE TANGENT AT THE SAME POINT OF AB.*

BY PROF. W. W. BEMAN, ANN ARBOR, MICHIGAN.

First part. Geometrical Demonstration.—The triangle CMN being isosceles, $\angle CMN = \angle CNM$. $\therefore \angle NMD = \angle MND' = \angle NSD'$; $\therefore$ the triangles QMD and QSD are similar, and $MD : SD :: r : R :: QD : QD'$. By division, $(R-r) : R :: DD' : QD'$, or $(R-r) : R :: (R+r) : QD'$;

$$\therefore QD' = \frac{R(R+r)}{R-r},$$

i. e. QD' is constant, and M-N in all positions will pass through Q. Similarly it may be shown that NM' in all positions will pass through the point Q'.



$$QO = \frac{2Rr}{R-r}, \qquad Q'O = \frac{2Rr}{R+r}.$$

Analytical Demonstration.—Let O be the origin, QD' and BC the axes, (x_1, y_1) coordinates of M or M', $(x_2, y_2,)$ coordinates of N, $(0, a)$ coordinates of the variable point C. Then we easily obtain

$$y_1^2 = 2rx_1 - x_1^2 \quad (1), \qquad y_2^2 = 2Rx_2 - x_2^2; \qquad (2)$$

$$yy_1 = r(x+x_1) - xx_1 \quad (3), \qquad yy_2 = R(x+x_2) - xx_2; \qquad (4)$$

$$y - y_1 = \frac{y_1 - y_2}{x_1 - x_2}(x - x_1). \qquad (5)$$

Since C $(0, a)$ is a point in both (3) and (4), we have

$$ay_1 = rx_1 \quad (6), \qquad ay_2 = Rx_2. \qquad (7)$$

Combining (6) with (1) and (7) with (2), we find

$$x_1 = \frac{2a^2r}{a^2+r^2},\ y_1 = \frac{2ar^2}{a^2+r^2},\ x_2 = \frac{2a^2R}{a^2+R^2},\ y^2 = \frac{2aR^2}{a^2+R^2}.$$

Substituting these values in (5), and reducing,

$$a^4y - a^3[(r+R)x - 2rR] + a^2r(r-R)y - ar^2[(r+R)x - 2rR] - r^3Ry = 0. \quad (8)$$

This locus evidently passes through the point $[2rR \div (r+R),\ 0]$ for *all* values of a; $\therefore$ this point is the envelope required. r is *positive* when the circles are tagent internally, *negative* when they are tangent externally.

Second part. To find the lacus of the middle point of MN.—Letting ρ_1 and ω be the polar coordinates of any point in the circle D'', the pole being at Q', we have

$$\rho_1 = -r\left\{\frac{R-r}{R+r}\cos\omega \pm \frac{\sqrt{[(R+r)^2 - (R-r)^2\sin^2\omega}}{R+r}\right\} \qquad (1)$$

Again, letting ρ_2 and ω be the polar coordinates of any point in the circle D', the pole being at Q', we have

$$\rho_2 = R\left\{\frac{R-r}{R+r}\cos\omega \pm \frac{\sqrt{[(R+r)^2 - (R-r)^2\sin^2\omega}}{R+r}\right\}. \qquad (2)$$

Now, if ρ and ω be the coordinates of any point in the locus desired, we easily get from (1) and (2),

$$\rho = \tfrac{1}{2}\left\{\frac{(R-r)^2}{R+r}\cos\omega \pm \sqrt{[(R+r)^2 - (R-r)^2\sin^2\omega}\right\}. \qquad (3)$$

one form of the equation.

If the rectangular equation with origin at O be required, we make

$$\rho = \frac{\sqrt{(R+r)^2y^2 + [(R+r)x - 2Rr]^2}}{R+r},\ \sin\omega = \frac{(R+r)y}{\sqrt{(R+r)^2y^2 + [(R+r)x - 2Rr]^2}}$$

$$\cos\omega = \frac{(R+r)x - 2Rr}{\sqrt{(R+r)^2y^2 + [(R+r)x - 2Rr]^2}},$$ and get, after some labor,

$(R+r)^2(y^2+x^2)^2 - (R+r)(R^2+6Rr+r^2)x(x^2+y^2) + (R+r)^2Rry^2 + 4Rr \times (R^2+3Rr+r^2)x^2 - 4R^2r^2(R+r)x = 0$, r being regarded as *positive* when the circles are tangent internally, and *nenative* when tangent externally. We may write it $(R \pm r)^2(y^2+x^2)^2 - (R \pm r)(R^2 \pm 6Rr + r^2)x(x^2+y^2) \pm (R \pm r)^2Rry^2 \pm 4Rr(R^2 \pm 3Rr + r^2)x^2 - 4R^2r^2(R \pm r)x = 0$, the upper sign being taken when the circles are tang't internally, and the lower when tang't ext'y.